Multiple-Criteria Fuzzy Evaluation: The FuzzME Software Package

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Abstract— *This paper introduces a new software product FuzzME. It was developed as a tool for creating fuzzy models of multiple-criteria evaluation and decision making. The type of evaluations employed in the fuzzy models fully corresponds with the paradigm of the fuzzy set theory; the evaluations express the (fuzzy) degrees of fulfillment of corresponding goals. The FuzzME software takes advantage of linguistic fuzzy modeling to the maximum extent. In the FuzzME software, both quantitative and qualitative criteria can be used. The basic structure of evaluation is described by a goals tree. Within the goals tree, aggregation of partial fuzzy evaluations is done either by one of fuzzified aggregation operators (fuzzy weighted average, ordered fuzzy weighted average) or by a fuzzy expert system.*

The paper contains an illustrative example of the software usage. The application concerns a soft-fact-rating problem that was solved in one of Austrian banks.

Keywords— Fuzzy expert system, Fuzzy OWA operator, Fuzzy weighted average, Multiple-criteria fuzzy evaluation, Normalized fuzzy weights, Software.

1 Introduction

In practice, sophisticated models of multiple-criteria evaluation are required (e.g. rating of clients in banks, evaluation of hospitals or universities, comparison of alternative solutions to ecological problems). For creating the evaluating models, setting some of their input data and interpreting their outputs, expert's knowledge is needed (e.g. evaluations of alternatives according to qualitative criteria, partial evaluating functions for quantitative criteria, a choice of a suitable type of aggregation, criteria weights, or eventually, a rule base describing the relation between criteria values, the overall evaluation and a linguistic description of obtained results). Because uncertainty is the typical feature of any expert information, the fuzzy set theory is a suitable mathematical tool for creating such models. For the practical use of the fuzzy models of multiple-criteria evaluation, their user-friendly software implementation is necessary. But a good theoretical basis of the used models is crucial, too. The clear and well-elaborated theory of multiple-criteria fuzzy evaluation makes it possible to create an understandable methodics for the software user. And a good methodics is essential for correct application of any software to solving real problems.

There is a large number of papers and books dealing with the theory and methods of multiple-criteria evaluation that make use of the fuzzy approach (e.g. [1], [2], [3], [4]). Multiple-criteria evaluation (as a basis of decision making) was even one of the earliest applications of fuzzy sets (see [1]). In more than 40 years of existence of the fuzzy sets theory, several software products for multiple-criteria decision making, which use the fuzzy modeling principles in different degrees and in different ways, have been developed. In practice, FuzzyTECH (see [5]) is probably the best-known of these. It enables to use the specific instruments of the fuzzy set theory for solving multiple-criteria evaluation and decision making problems. Generally, FuzzyTECH is a universal software product which makes it possible to create and use fuzzy expert systems (or fuzzy controllers). It also includes the possibility to derive fuzzy rule bases from given data by means of neural network algorithms. In the book [6], there were described many successful applications of this software to solving multiple-criteria evaluation and decision making problems in the areas of business and finance. Similarly, fuzzy toolboxes of general mathematical systems such as Matlab can be used for multiple-criteria decision making.

The FuzzME software (**Fuzz**y models of **M**ultiple-criteria **E**valuation), presented in this paper, is based on a theoretical concept of evaluation which is very close to the original Zadeh's ideas. Similarly to his paper [1], the evaluations of alternatives according to particular criteria represent their degrees of fulfillment of the corresponding partial goals. Besides evaluations expressed by real numbers in $[0, 1]$, fuzzy evaluations modeled by fuzzy numbers on the same interval are employed in the software. They represent, analogously, the fuzzy degrees of fulfillment of the partial goals which are connected to the criteria. Resulting fuzzy evaluations, which are obtained by aggregation, have a similar clear interpretation. This theoretical approach to (fuzzy) evaluation was published in the book [7] and in the paper [8].

The predecessor of the FuzzME software package in terms of the used theoretical basis was the NEFRIT software. This software for multiple-criteria evaluation and decision making, which is also based on fuzzy technologies, was developed in about 2000 by the Czech software company TESCO SW Inc. The fuzzy model of evaluation applied there is described in detail in [8] and in the book [7] (a demo version of NEFRIT is enclosed in the book). NEFRIT makes it possible to work with expert fuzzy evaluations of alternatives according to qualitative criteria. Values of the quantitative criteria can either be crisp or fuzzy. Evaluating functions for quantitative criteria represent membership functions of corresponding partial goals. The main evaluation structure is expressed by a goals tree. For aggregation of the partial fuzzy evaluations the weighted average method is used. The weights (crisp only, not fuzzy) express the shares of partial evaluations in the aggregated one. Fuzzy evaluations on all levels of aggregation express the fulfillment of the corresponding goals. The NEFRIT software was originally developed for the Czech National Bank (decision making about granting a credit). Further, it was used e.g. by the Czech Tennis Association, the Czech Basketball Association and in other institutions. Nowadays it is tested by the Supreme Audit Office of the Czech Republic.

In contrast to NEFRIT, the FuzzME software makes it possible to use also uncertain weights in the aggregation by means of the weighted average method. The theory of normalized fuzzy weights, procedures for their setting, and an effective algorithm for calculation of fuzzy weighted average are taken from [9], [10] and [11]. Another fuzzy aggregation operator, available in the FuzzME software, is a fuzzified OWA operator. Again, it works with normalized fuzzy weights. The fuzzy OWA operator and the used algorithm for its calculation are described in [12]. In the FuzzME software, multiple-criteria evaluating functions can also be defined by means of fuzzy rule bases. In accordance with [7], two algorithms are offered for the approximate reasoning - the standard Mamdani algorithm and a modified Sugeno algorithm.

There are also software tools for multiple-criteria decision making based on other mathematical methods. But they are usually designed for solving a particular assignment. Our investigation by means of Internet did not result software fully comparable to FuzzME. Its universality and comprehensiveness make it unique.

2 Preliminaries

A fuzzy set A on a universal set X is characterized by its membership function $A : X \to [0, 1]$. $Ker\ A$ denotes a kernel of A, $Ker\ A = \{x \in X \mid A(x) = 1\}$. For any $\alpha \in [0, 1]$, A_α denotes an α-cut of A, $A_\alpha = \{x \in X \mid A(x) \geq \alpha\}$. A support of A is defined as $Supp\ A = \{x \in X \mid A(x) > 0\}$. The symbol $hgt\ \ A$ denotes a height of the fuzzy set A, $hgt\ A = \sup \{A(x) \mid x \in X\}$. An intersection and a union of the fuzzy sets A and B on X are defined for all $x \in X$ by the following formulas: $(A \cap B)(x) = \min \{A(x), B(x)\}$, $(A \cup B)(x) = \max \{A(x), B(x)\}$.

A fuzzy number is a fuzzy set C on the set of all real numbers $\Re$ which satisfies the following conditions: a) the kernel of C, $Ker\ C$, is not empty, b) the α-cuts of C, C_α, are closed intervals for all $\alpha \in (0, 1]$, c) the support of C, $Supp\ C$, is bounded. A fuzzy number C is called to be defined on $[a, b]$, if $Supp\ C \subseteq [a, b]$. Real numbers $c^1 \leq c^2 \leq c^3 \leq c^4$ are called significant values of the fuzzy number C if the following holds: $[c^1, c^4] = Cl(Supp\ C), [c^2, c^3] = Ker\ C$, where $Cl(Supp\ C)$ denotes a closure of $Supp\ C$.

Any fuzzy number C can be characterized by a pair of functions $\underline{c} : [0, 1] \to \Re$, $\overline{c} : [0, 1] \to \Re$ which are defined by the following formulas: $C_\alpha = [\underline{c}(\alpha), \overline{c}(\alpha)]$ for all $\alpha \in (0, 1]$, and $Cl(Supp\ C) = [\underline{c}(0), \overline{c}(0)]$. The fuzzy number C is called to be linear if both the functions $\underline{c}$, $\overline{c}$ are linear. A linear fuzzy number is fully determined by its significant values because $\underline{c}(\alpha) = (c_2 - c_1) \cdot \alpha + c_1$, $\overline{c}(\alpha) = (c_3 - c_4) \cdot \alpha + c_4$. For that reason, we can denote it as $C = (c^1, c^2, c^3, c^4)$.

An ordering of fuzzy numbers is defined as follows: a fuzzy number C is greater than or equal to a fuzzy number D, if $C_\alpha \geq D_\alpha$ for all $\alpha \in (0, 1]$.

A fuzzy scale makes it possible to represent a closed interval of real numbers by a finite set of fuzzy numbers. Let $T_1, T_2, ..., T_s$ be fuzzy numbers defined on $[a, b]$, forming a fuzzy partition on the interval, i.e., for all $x \in [a, b]$ the following holds

$$\sum_{i=1}^{s} T_i(x) = 1, \tag{1}$$

then the set of the fuzzy numbers can be linearly ordered (see [7]). If the fuzzy numbers $T_1, T_2, ..., T_s$ are defined on $[a, b]$, form a fuzzy partition on the interval and are numbered according to their linear ordering, then they are said to form a fuzzy scale on $[a, b]$.

An uncertain division of the whole into m parts can be modeled by normalized fuzzy weights. Fuzzy numbers $V_1, ..., V_m$ defined on $[0, 1]$ are normalized fuzzy weights if for any $i \in \{1, ..., m\}$ and any $\alpha \in (0, 1]$ it holds that for any $v_i \in V_{i\alpha}$ there exist $v_j \in V_{j\alpha},\ j = 1, ..., m,\ j \neq i$, such that

$$v_i + \sum_{j=1, j \neq i}^{m} v_j = 1. \tag{2}$$

3 The FuzzME software

The mathematical models of the FuzzME software are based primarily on the theory and methods of multiple-criteria evaluation that were published in [7] and [8]. The theory of normalized fuzzy weights, the definition of fuzzy weighted average, and the algorithm for its computation were taken from [9], [10] and [11]. The fuzzified OWA operator and the algorithm for its calculation published in [12] are also used in the software.

In the FuzzME software, the basic structure of the fuzzy model of multiple-criteria evaluation is expressed by a goals tree. The root of the tree represents the overall goal of evaluation and each branch corresponds to a partial goal. The goals at the ends of branches are connected either with quantitative or qualitative criteria.

When an alternative is evaluated, evaluations with respect to criteria connected with the terminal branches are calculated first. Independently of the criterion type, each of the evaluations is described by a fuzzy number defined on the interval $[0, 1]$. It expresses the fuzzy degree of fulfillment of the corresponding partial goal.

These partial fuzzy evaluations are then aggregated according to the defined type of the tree node. Three types of aggregation are available: a fuzzy weighted average (fuzzy WA), an ordered fuzzy weighted average (fuzzy OWA) or aggregation by means of a fuzzy expert system. For aggregation by fuzzy weighted average or ordered fuzzy weighted average, normalized fuzzy weights must be set. The weights express uncertain shares of the partial evaluations in the aggregated one. For the fuzzy expert system, the fuzzy rule base must be defined and an inference algorithm must be chosen (either the Mamdani algorithm or the generalized Sugeno algorithm of approximate reasoning).

The overall evaluation reflects the degree of fulfillment of the main goal. A verbal description of the overall evaluation can be obtained by means of the implemented linguistic approximation algorithm.

The overall evaluations can be compared within the frame of a given set of alternatives. By this comparison the best of the alternatives can be chosen. That is why the FuzzME software can be also used as a decision support system.

The import and export of data is supported by the software, too. The FuzzME software is available in the Czech and English versions.

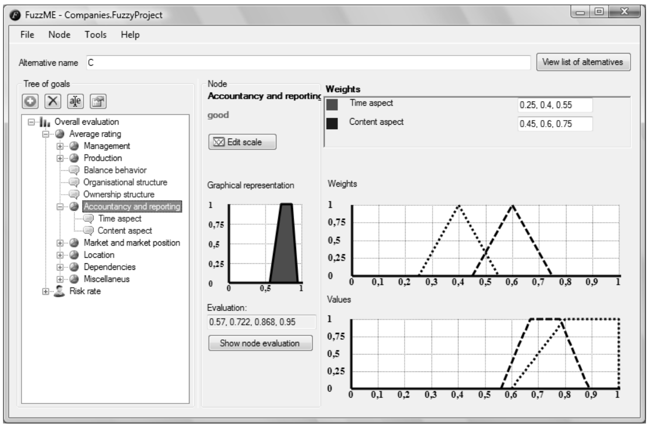

Figure 1: The main window of the software

3.1 *Goals tree*

Goals trees represent the basic structure of fuzzy models of multiple-criteria evaluation in the FuzzME software. When a goals tree is designed, the main goal is consecutively divided into goals of progressively lower levels. The process of division is stopped when such goals are reached whose fulfillment can be assessed by means of some known characteristics of alternatives (i.e. quantitative or qualitative criteria).

The design of a tree structure in the goals-tree editor is the first step in forming a fuzzy evaluation model in FuzzME. In the next step, the type of each node in the tree must be specified. For the nodes at the ends of tree branches the user defines if the node is connected with a quantitative or qualitative criterion. For the other nodes he/she sets the type of aggregation - fuzzy weighted average, ordered fuzzy weighted average or fuzzy expert system. An example of a goals tree is illustrated in Fig. 2.

3.2 *Criteria of evaluation*

In the models of evaluation created by the FuzzME software, qualitative and quantitative criteria can be combined arbitrarily.

3.2.1 *Qualitative criteria*

According to qualitative criteria, alternatives are evaluated verbally, by means of values of linguistic variables of special kinds - linguistic scales, extended linguistic scales and linguistic scales with intermediate values.

A linguistic variable is defined as a quintuple $(\mathcal{V}, \mathcal{T}(\mathcal{V}), X, G, M)$, where $\mathcal{V}$ is a name of the variable, $\mathcal{T}(\mathcal{V})$ is a set of its linguistic values, X is a universal set on which the meanings of the linguistic values are defined, G is a syntactic rule for generating values in $\mathcal{T}(\mathcal{V})$, and M is a semantic rule which maps each linguistic value $\mathcal{C} \in \mathcal{T}(\mathcal{V})$ to its mathematical meaning, $C = M(\mathcal{C})$, which is a fuzzy set on X.

A linguistic scale on $[a, b]$ is a special case of the linguistic variable $(\mathcal{V}, \mathcal{T}(\mathcal{V}), X, G, M)$, where $X = [a, b]$, $\mathcal{T}(\mathcal{V}) = \{\mathcal{T}_1, \mathcal{T}_2, ..., \mathcal{T}_s\}$ and the meanings of the linguistic values $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_s$ are modeled by fuzzy numbers $T_1, T_2, \ldots, T_s$ which form a fuzzy scale on $[a, b]$. As the set of linguistic values of the scale is defined explicitly, it is not necessary to include the grammar G into the scale notation.

In the FuzzME software, the user defines a linguistic scale for each qualitative criterion in the fuzzy-scale editor. For example, the linguistic scale *quality of a product* can contain linguistic values *poor*, *substandard*, *standard*, *above standard* and *excellent*. The evaluating linguistic scale is usually defined on $[0, 1]$; in other cases, it has to be transformed to this interval.

The extended linguistic scale contains, besides elementary terms of the original scale, $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_s$, also derived terms in the form $\mathcal{T}_i$ *to* $\mathcal{T}_j$, where $i < j, i, j \in \{1, 2, \ldots, s\}$. For example, the user can evaluate *quality of a product* by the linguistic term *standard to excellent*. The meaning of the linguistic value $\mathcal{T}_i$ *to* $\mathcal{T}_j$ is modeled by $T_i \cup_L T_{i+1} \cup_L \cdots \cup_L T_j$, where $\cup_L$ denotes the union of fuzzy sets based on the Lukasiewicz disjunction; e.g. $(T_i \cup_L T_{i+1})(x) = \min\{1, T_i(x) + T_{i+1}(x)\}$ for all $x \in \Re$.

The linguistic scale with intermediate values is the original linguistic scale enriched with derived terms *between* $\mathcal{T}_i$ *and* $\mathcal{T}_{i+1}$, $i \in \{1, 2, \ldots, s-1\}$. The meaning of the derived term *between* $\mathcal{T}_i$ *and* $\mathcal{T}_{i+1}$ is modeled by the arithmetic average of the fuzzy numbers T_i and T_{i+1}.

In the FuzzME software, the user evaluates a given alternative according to a qualitative criterion by selecting a proper linguistic evaluation from a drop-down list box. He/she can choose the value from a standard linguistic scale, extended scale or scale with intermediate values.

The three mentioned structures of linguistic values are also applied when resulting fuzzy evaluations are approximated linguistically.

3.2.2 *Quantitative criteria*

The evaluation of an alternative with respect to a quantitative criterion is calculated from the measured value of the criterion by means of the evaluating function expertly defined for the criterion. The evaluating function is the membership function of the corresponding partial goal. The FuzzME software admits both crisp and fuzzy values of quantitative criteria. The fuzzy values represent inaccurate measurements or expert estimations of the criteria values. In the case of a fuzzy value, the corresponding partial fuzzy evaluation is calculated by the extension principle.

In the FuzzME software, the evaluating function of a quantitative criterion is formally set by means of a fuzzy number. For example, if the evaluating function is defined by a linear fuzzy number $F = (f_1, f_2, f_3, f_4)$, then f_1 is the lower limit of all at least partly acceptable values of the criterion, f_2 is the lower limit of its fully satisfactory values, f_3 is the upper limit of the fully satisfactory values, and f_4 is the upper limit of the acceptable values.

For example, when a bank evaluates expected profitability of projects, the evaluating function can be defined by a linear fuzzy number with significant values 10, 30, 500, 500. In that case, values lower than 10% are not satisfying at all (the client would not be able to pay the money back to the bank). For the values from 10% to 30% the satisfaction of the bank is growing linearly. Values greater than 30% are fully satisfactory from the bank's point of view. Values greater than 500% are not supposed to occur. This way we can define a monotonous evaluating function, which is the most common in the evaluating models, by a fuzzy number.

3.3 *Methods of aggregation of partial evaluations*

The calculated partial fuzzy evaluations are then consecutively aggregated according to the structure of the goals tree. With respect to the defined type of the tree node, the fuzzy weighted average method, the ordered fuzzy weighted average method or the fuzzy expert system method is used for the aggregation. Each of the aggregation methods is suitable for a different situation:

The fuzzy weighted average is used if the goal corresponding with the node of interest is fully decomposed into disjunctive goals of the lower level. The normalized fuzzy weights represent uncertain shares of these lower-level goals in the goal corresponding with the considered node.

Again, the ordered fuzzy weighted average requires that the goal corresponding with the given node is decomposed into disjunctive goals of the lower level. In contrast to the fuzzy weighted average, the usage of this aggregation operator supposes special user's requirements concerning the structure of partial fuzzy evaluations. The normalized fuzzy weights again represent uncertain shares of the partial evaluations in the aggregated one. But the normalized fuzzy weights are not linked to the individual partial goals; the correspondence between the weights and the partial evaluations is given by the ordering of partial evaluations of the alternative of interest. It means, evaluations with respect to the same partial goal can have different weights for different alternatives.

If the relationship between the evaluations of the lower level and the evaluation corresponding with the given node is more complex (if neither of the two previous methods can be used), and if expert knowledge about the relationship is available, then the aggregation function is described by a fuzzy rule base of a fuzzy expert system. The approximate reasoning is used to calculate the resulting evaluation. In particular, evaluating function described by a fuzzy expert system is used if the fulfillment of a goal at the end of a tree branch depends on several mutually dependent criteria (i.e., if combinations of criteria values bring synergic or disynergic effects to the resulting multiple-criteria evaluation).

3.3.1 *Fuzzy weighted average*

If the fuzzy weighted average is used for aggregation of partial fuzzy evaluations, then the uncertain weights of the corresponding partial goals, which express their shares in the superior goal, must be set. To define consistent uncertain weights, a special structure of fuzzy numbers, normalized fuzzy weights, must be used.

In the FuzzME software, both real and fuzzy normalized weights can be used. Normalized real weights, i.e., real numbers $v_1, ..., v_m$, $v_j \geq 0$, $j = 1, ..., m$, $\sum\limits_{j=1}^{m} v_j = 1$, represent a special case of the normalized fuzzy weights.

The fuzzy weighted average of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, ..., U_m$ defined on $[0, 1]$, with the normalized fuzzy weights $V_1, \ldots, V_m$, is a fuzzy number U on $[0, 1]$ whose membership function is defined for any $u \in [0, 1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), ..., V_m(v_m), U_1(u_1), ..., U_m(u_m)\} \mid \sum_{i=1}^{m} v_i u_i = u,\ \sum_{i=1}^{m} v_i = 1,\ v_i, u_i \in [0, 1], i = 1, ..., m\}. \tag{3}$$

For an expert who sets the fuzzy weights, it is not so easy to satisfy the condition of normality. That is why the FuzzME software allows to set only an approximation to the normalized fuzzy weights - fuzzy numbers $W_1, ..., W_m$ on $[0, 1]$ satisfying the following weaker condition

$$\exists w_i \in Ker\ W_i,\ i = 1, ..., n : \sum_{i=1}^{n} w_i = 1. \tag{4}$$

The software removes the potential inconsistence in $W_1, ..., W_m$ and derives the normalized fuzzy weights $V_1, ..., V_m$ from them.

The structure of normalized fuzzy weights and the fuzzy weighted average operation are studied in detail in [9], [10] and [11]. Conditions for verifying normality of fuzzy weights, an algorithm for normalization of fuzzy weights satisfying the condition (4), and an algorithm for calculating fuzzy weighted average, which are all used in the FuzzME software, can be found there. Let us notice, that the used algorithm of fuzzy weighted average calculation is very effective.

3.3.2 *Ordered fuzzy weighted average*

The fuzzy OWA operator is used in case that the evaluator has special requirements concerning the structure of the partial evaluation. For example, he/she does not want any partial goal to be satisfied poorly. Then the weight of the minimum partial evaluation of any alternative equals 1, and the weights of all its other partial evaluations equal 0. The aggregated fuzzy evaluations then represent the guaranteed fuzzy degrees of fulfillment of all the partial goals (the fuzzy MINIMAX method). Another example of the fuzzy OWA operator usage could be the evaluation of subjects who can choose in which of the three areas they will be mostly involved. The evaluation algorithm should take into account their right of choice. Then, e.g., the results in the area where the subject performs best contribute to the overall evaluation by about one half, results from the second area by one third and results from the area in which the subject was least involved contribute to the overall evaluation only by one sixth. A practical application of such a fuzzy evaluation model could be the overall evaluation of the academic staff with respect to their results in the areas of research, education, and management of education and science.

The ordered fuzzy weighted average represents a fuzzification of the crisp OWA operator by means of the extension principle. Uncertain weights are modeled by normalized fuzzy weights as in the case of fuzzy weighted average.

The following notation will be used to define the ordered fuzzy weighted average: if $(x_1, ..., x_m)$ is a vector of real numbers, then $(x^{(1)}, ..., x^{(m)})$ is a vector in which for all $j \in \{1, \ldots, m\}$, $x^{(j)}$ is the j-th greatest number of $x_1, ..., x_m$.

The ordered fuzzy weighted average of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, ..., U_m$ defined on

$[0,1]$, with the normalized fuzzy weights $V_1, \ldots, V_m$, is a fuzzy number U on $[0,1]$ whose membership function is defined for any $u \in [0,1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), ..., V_m(v_m), U_1(u_1), ..., U_m(u_m)\} \mid \sum_{i=1}^{m} v_i u^{(i)} = u,\ \sum_{i=1}^{m} v_i = 1,\ v_i, u_i \in [0,1], i = 1, ..., m\}. \quad (5)$$

The algorithm used to calculate the ordered fuzzy weighted average in the FuzzME software was taken from [12], where fuzzification of the OWA operator is described in detail. The used algorithm is an analogy to the one used for the fuzzy weighted average.

3.3.3 *Fuzzy expert system*

The fuzzy expert system is used if the relationship between the criteria (or the partial evaluations) and the overall evaluation is complicated. Theoretically, it is possible to model, with an arbitrary precision, any Borel measurable function by means of a fuzzy rule base (properties of Mamdani and Sugeno fuzzy controllers, see e.g. [13]) In reality, the quality of the approximation is limited by the expert's knowledge of the relationship.

If the fuzzy rule base models the relation between values of criteria and the fulfillment of the corresponding partial goal, then the evaluation function is of the following form

$$\begin{array}{l} \text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{1,1} \text{ and } \ldots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{1,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_1 \\ \text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{2,1} \text{ and } \ldots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{2,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{n,1} \text{ and } \ldots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{n,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_n \end{array} \quad (6)$$

where for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $(\mathcal{C}_j, \mathcal{T}(\mathcal{C}_j), V_j, M_j)$ are linguistic scales representing the criteria, $\mathcal{A}_{i,j} \in \mathcal{T}(\mathcal{C}_j)$ are their linguistic values, $(\mathcal{E}, \mathcal{T}(\mathcal{E}), [0,1], M_e)$ is a linguistic scale representing the evaluation of alternatives and $\mathcal{U}_i \in \mathcal{T}(\mathcal{E})$ are its linguistic values.

In the FuzzME software, rule bases are defined expertly. The user defines such a rule base by assigning a linguistic evaluation to each possible combination of linguistic values of criteria.

For given values of criteria, a resulting fuzzy evaluation is calculated either by the Mamdani fuzzy inference algorithm or by the generalized Sugeno inference.

In the case of the Mamdani fuzzy inference, the degree h_i of correspondence between the given m-tuple of fuzzy values $(A_1^{'}, A_2^{'}, \ldots, A_m^{'})$ of criteria and the mathematical meaning of the left-hand side of the *i*-th rule is calculated for any $i = 1, \ldots, n$ in the following way

$$h_i = \min\{hgt(A_1^{'} \cap A_{i,1}), \ldots, hgt(A_m^{'} \cap A_{i,m})\}. \quad (7)$$

Then for each of the rules, the output fuzzy value $U_i^{'}$, $i = 1, \ldots, n$, corresponding to the given input fuzzy values, is calculated as follows

$$\forall y \in [0,1] : U_i^{'}(y) = \min\{h_i, U_i(y)\}. \quad (8)$$

The final fuzzy evaluation of the alternative is given as the union of all the fuzzy evaluations that were calculated for the particular rules in the previous step, i.e.,

$$U^{'} = \bigcup_{i=1}^{n} U_i^{'}. \quad (9)$$

Generally, the result obtained by the Mamdani inference algorithm need not be a fuzzy number. So, for further calculations within the fuzzy model, it must be approximated by a fuzzy number.

The advantage of the generalized Sugeno inference algorithm (see [7]) is that the result is always a fuzzy number. In its first step, the degrees of correspondence h_i, $i = 1, \ldots, n$, are calculated in the same way as in the Mamdani fuzzy inference algorithm.

The resulting fuzzy evaluation U is then computed as a weighted average of the fuzzy evaluations U_i, $i = 1, 2, \ldots, n$, which model the mathematical meanings of linguistic evaluations on the right-hand sides of the rules, with the weights h_i. This is done by the following formula

$$U = \frac{\sum_{i=1}^{n} h_i . U_i}{\sum_{i=1}^{n} h_i}. \quad (10)$$

3.4 *Overall fuzzy evaluations, the optimum alternative*

The final result of the consecutive aggregation of the partial fuzzy evaluations is an overall fuzzy evaluation of the given alternative. The obtained overall fuzzy evaluations are fuzzy numbers on $[0,1]$. They express uncertain degrees of fulfillment of the main goal by the particular alternatives.

The FuzzME software compares alternatives according to the centers of gravity of their overall fuzzy evaluations. A center of gravity of a fuzzy number U on $[0,1]$ that is not a real number, is defined as follows

$$t_U = \frac{\int_0^1 U(x).x\ dx}{\int_0^1 U(x)\ dx}. \quad (11)$$

If $U = u$ and $u \in \Re$, then $t_U = u$. In the FuzzME software, the optimum alternative is the one whose overall fuzzy evaluation has the largest center of gravity.

At present, the FuzzME software is aimed above all at solving multiple-criteria evaluation problems. To ensure high performance in choosing the optimum alternative, it will be necessary to include in the software other methods of ordering of the fuzzy evaluations in the future. Some approaches are proposed in [7] and further research in this area is planned.

4 Example of a practical application of the FuzzME software

The FuzzME software was tested e.g. on a soft-fact-rating problem of one of the Austrian banks. The problem was solved in co-operation with the Technical University in Vienna (see [14]). The fuzzy model of evaluation represents a part of the creditability evaluation of companies carried out by the bank - the evaluation according to soft (qualitative) data, which complements the evaluation according to hard (quantitative) data. The previous practical experience of the bank showed that it is not good to restrict the evaluation to hard data only.

In total, 62 companies were evaluated by the fuzzy model. The goals tree of the model contained 27 qualitative criteria.

During the testing, two approaches were compared - the original soft-fact-rating model used by the bank and the fuzzy models created in the FuzzME software.

The original evaluation model used simple discrete numeric scales with intuitively set linguistic descriptors for the evaluation according to the particular qualitative criteria. The aggregation of partial evaluation was done by the standard weighted average.

In testing by the FuzzME software, the applied linguistically described numeric scales were analyzed. It was found that in some cases the correspondence between the linguistic and numerical values was not perfect. Two new fuzzy models were formed. The first one used uniform fuzzy scales representing a simple fuzzification of the original numeric scales. The other worked with fuzzy values which tried to model, as closely as possible, the linguistic descriptors used in the original evaluation model. The results of the two models were quite different. At the same time, the normalized crisp weights were replaced by normalized fuzzy weights which correspond better to the expert's knowledge about the importance of criteria.

The subsequent discussion on results of the soft-fact-rating showed that there exist criteria values and combinations of criteria values which signalize a substantial danger that the company will go bankrupt or at least will have problems acquitting the debt. That is why, besides the evaluation of companies based on fuzzy weighted average ("average rating"), a fuzzy expert system was applied to calculate another evaluation ("a risk rate of the company"). The particular rules of the base identified the dangerous combinations of criteria values and assigned to them the corresponding risk rates. The solely use of the original fuzzy model without the fuzzy expert system would have lead to a rating score, which may have underestimated the risk inherent to this company. The use of the fuzzy expert systems offers the possibility to visualize and calculate such additional risk combinations.

Finally, both evaluations were aggregated with the fuzzy MINIMAX method. This method is a special case of a fuzzy OWA operator.The resulting evaluation is the infimum of the fuzzy numbers representing the partial evaluations.

The obtained results showed that the solid theoretical basis of the evaluation fuzzy models formed in the FuzzME software improves the quality of evaluations. Positive experiences with such fuzzy models of evaluation could win over the present-day opponents to the soft-fact-rating in the future.

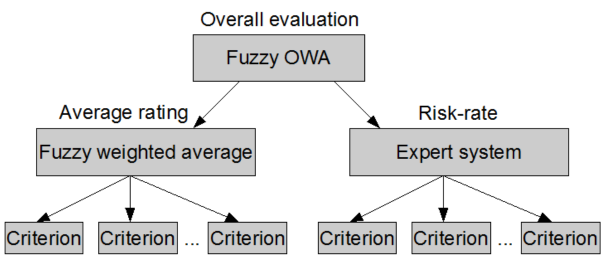

Figure 2: The simplified structure of the used goals tree

5 Conclusion

The software product FuzzME is a result of many years of research in the area of the theory and methods of multiple-criteria fuzzy evaluation. The type of evaluation consistently used in the software corresponds well to the fuzzy sets theory paradigm; the evaluations of alternatives express the fuzzy degrees of fulfillment of given goals. In the FuzzME software, several new methods, algorithms and tools of fuzzy modeling were implemented, e.g.: a structure of normalized fuzzy weights, fuzzy weighted average and ordered fuzzy weighted average operations and algorithms for their calculation, linguistic scales and linguistic variables derived from them. Well-elaborated theoretical basis of the FuzzME software provides a clear interpretation of all steps of the evaluation process and brings understanding of methodology to the user. The FuzzME software is user-friendly. The positive features of the software product proved themselves by solving the mentioned soft-fact-rating problem.

References

[1] R.E. Bellman and L.A Zadeh. Decision-making in fuzzy enviroment. *Management Sci.*, 17 (4):141–164, 2007.

[2] R.R. Yager. On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Trans.Systems Man Cybernet*, 3 (1):183–190, 1988.

[3] Y. J. Lai and C. L. Hwang. *Multiple Objective Decision Making.* Springer - Verlag, Berlin, Heidelberg, 1994.

[4] H. Rommelfanger. *Fuzzy Decision Support Systeme.* Springer - Verlag, Berlin, Heidelberg, 1988.

[5] INFORM GmbH. FuzzyTECH home page. `http://www.fuzzytech.com/`.

[6] C. Von Altrock. *Fuzzy Logic and NeuroFuzzy Applications in Bussiness and Finance*. Prentice Hall, New Jersey, 1996.

[7] J. Talašová. *Fuzzy methods of multiple criteria evaluation and decision making (in Czech).* Publishing House of Palacký University, Olomouc, 2003.

[8] J. Talašová. NEFRIT - multicriteria decision making based on fuzzy approach. *Central European Journal of Operations Research*, 8(4):297–319, 2000.

[9] O. Pavlačka. *Fuzzy methods of decision making (in Czech).* PhD thesis, Faculty of Science, Palacký Univerzity, Olomouc, 2007.

[10] O. Pavlačka and J. Talašová. Application of the fuzzy weighted average of fuzzy numbers in decision-making models. *New Dimensions in Fuzzy Logic and related technologies, Proceedings of the 5th EUSFLAT Conference, Ostrava, Czech Republic, September 11-14 2007, (Eds. M. Štěpnička, V. Novák, U. Bodenhofer)*, II:455–462, 2007.

[11] O. Pavlačka and J. Talašová. The fuzzy weighted average operation in decision making models. *Proceedings of the 24th International Conference Mathematical Methods in Economics, 13th - 15th September 2006, Plzeň (Ed. L. Lukáš)*, pages 419–426, 2006.

[12] J. Talašová and I. Bebčáková. Fuzzification of aggregation operators based on Choquet integral. *Aplimat - Journal of Applied Mathematics*, 1(1):463–474, 2008. ISSN 1337-6365.

[13] B. Kosko. *Fuzzy thinking: The new science of fuzzy logic.* Hyperion, New York, 1993.

[14] K. Fürst. *Applying Fuzzy Models in Rating Systems.* Department of Statistics and Probability Theory, Vienna University of Technology, Vienna, 2008. Term paper.